SIMPLIFIED SOLUTION OF THE STRESSES IN SPACE FRAME STRUCTURES

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(a) "Simplified Solution of the Stresses in Space Frame Structures" by Lt. C. L. Hayen, CEC, USN. This thesis discusses the analysis of space frame structures by the method of "tension coefficients". A simplified application of the method is presented and two examples are presented to illustrate the application. The simplified application could be used in the analysis of simple space frame structures, however further research is considered necessary before more complex frames could be analyzed by this method.

SIMPLIFIED SOLUTION OF
SPACE FRAME STRUCTURES

Submitted to the faculty of
Rensselaer Polytechnic Institute in
partial fulfillment of the require-
ments for the degree of Master of
Civil Engineering.

By
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August, 1947
Troy, New York

Thesis
H4

ACKNOWLEDGEMENT

The author wishes to thank Professor Joseph S. Kinney and Professor John M. Beatty of the Civil Engineering Department of Rensselaer Polytechnic Institute for their suggestions and criticism in connection with this study.

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INTRODUCTION

Since all framed structures have length, breadth, and thickness, all frames actually are space structures. Designers are accustomed to treat designs of frames as a group of planar frames, despite the fact that the twisting action of an eccentric or diagonal wind force upon a steel frame produces a problem involving forces in space. There are structures where the entire analysis must be studied in three dimensions. Framed pedestals, towers with three or more legs, framed domes, and bridge trusses having a common chord, are examples of space frames. The necessary computations are not particularly complicated, but they are more tedious than those involved in the analysis of planar structures. Methods have been devised to reduce the tedious work involved in the solution of space frames. The subject of this paper is another simplification of the solution of space structures.

Determinancy of space structures. Methods for simplified space frame solutions have been investigated in the last fifteen years by many of the leading design engineers and mathematicians. These methods have been derived from work done in Europe during the nineteenth century. First solutions were regular calculations of forces and moments at joints in the three planes. The determinancy of the

structure is based on the fact that the three force and thre moment equations can be used at each joint which has members in three planes. Each joint with members in only one plane will reduce the total number of possible equations by two. According to Grinter, these statements give the formula for determining whether or not a frame is determinate. His equation:

$$r + b = 3j - p$$

where r = Number of reactions
b = Number of bars
j = Number of joints
p = Number of joints in which all members lie in one planes.

Spofford's equation for determinancy has a greater range for application. If a reaction is eliminated by method of construction either by fixing the direction of certain reactions or by eliminating it entirely, the number of equations should be reduced. Therefore, if a frame required a certain number of reactions for stability and this number gives an indeterminate structure, certain reactions are changed by construction to give a determinate structure. Spofford's equation:

$$3j = b + 3r - s$$

where j = Number of joints
r = Number of suppotts
b = Number of bars
s = Number of reactions eliminated

The Reaction Problem. These tow equations seem to tie down the possibility of solving a space frame. The problem which still remains however is to place the reactions and use enough to make the structure stable. It is evident that in many complex structures, the most difficult problem is to place the reaction so as to have a stable and determinate structure. In preparing this paper, the placing of reactions for the various illustrative problems presented the greatest difficulty. This was met in some cases by stating conditions which would limit the number of reactions. The reason for stating conditions will be further discussed for each problem.

Method of Tension Coefficients. A method of tension coefficients was decised by Professor R. V. Southwell of Oxford which simplifies the straight computations involved when the six equations are written at each joint. This method will be explained briefly because the subject of this paper is a modification of the method of tension coefficients.

In this method, the stress in a bar equals the product of the length of the member and the tension coefficient $T = tL$. Assume a bar AB in a space frame and carrying a tensile force of T_{AB}. If L_{AB} is the

length, the force on the bar is equal to $T_{AB} = t_{AB}L_{AB}$ If the coordinates in space of points A and B are respectively x_A, y_A, z_A and x_B, y_B, z_B, then the component of T_{AB} in the X direction in bar AB equals $T_{AB}(x_B-x_A)$. If this component is divided by L_{AB}, then it equals $t_{AB}(x_B-x_A)$. Also

$$\frac{T_{AB}(y_B-y_A)}{L_{AB}} = t_{AB}(y_B-y_A)$$

and

$$\frac{T_{AB}(z_B-z_A)}{L_{AB}} = t_{AB}(z_B-z_A)$$

Components along X, Y, and Z axes at B are:

$$t_{BA}(x_A-x_B)$$
$$t_{BA}(y_A-y_B)$$
$$t_{BA}(z_A-z_B)$$

which are equal in magnitude but opposite in sign to those at A. At any joint the summation of the tension coefficients times the projections on the axis plus external forces is equal to zero along each axis. Since all members are assumed to have tension, tension coffieients with positive signs are in tension, and those with negative signs are in compression.

<u>Object</u> The method of tension coefficients can be further simplified into a system of projection ratio multiples. The object of this paper is to develop the

theory and explanation of this method.

Comparison. This simplified solution is similar to the use of index numbers in planar structures. The solution is very simple for simple frames; however for complex frames, the solution is still complex. On comparison with other methods, it gives a simpler solutions for the simplex frames investigated; however it is probable that some problems could be solved easier by other methods. This could only be shown by comparison with every type and therefore is beyond the scope of this paper.

THE SIMPLIFIED SOLUTION

The method of tension coefficients can be modified to a system similar to index numbers in planar trusses. These numbers will be called ht numbers in this paper. The ht number in each bar will be positive if tensile stress and negative if compressive stress. The stress in the member will be a product of the ht number and the length of the member divided by the term h. The term h is the key to this simplified solution; therefore it must be understood what the distance h is before the problem can be solved by this method. In tension coefficients method, the solution is started at a joint which has an external load. There are one or more members which take this external load out of the joint. If the load is parallel to one of the axes chosen, the distance h is the distance from the first joint solved to the next joint on a line parallel to the load. The distance h for other directions of loading will be taken up in a later problem and a general definition of h will be given in the summary.

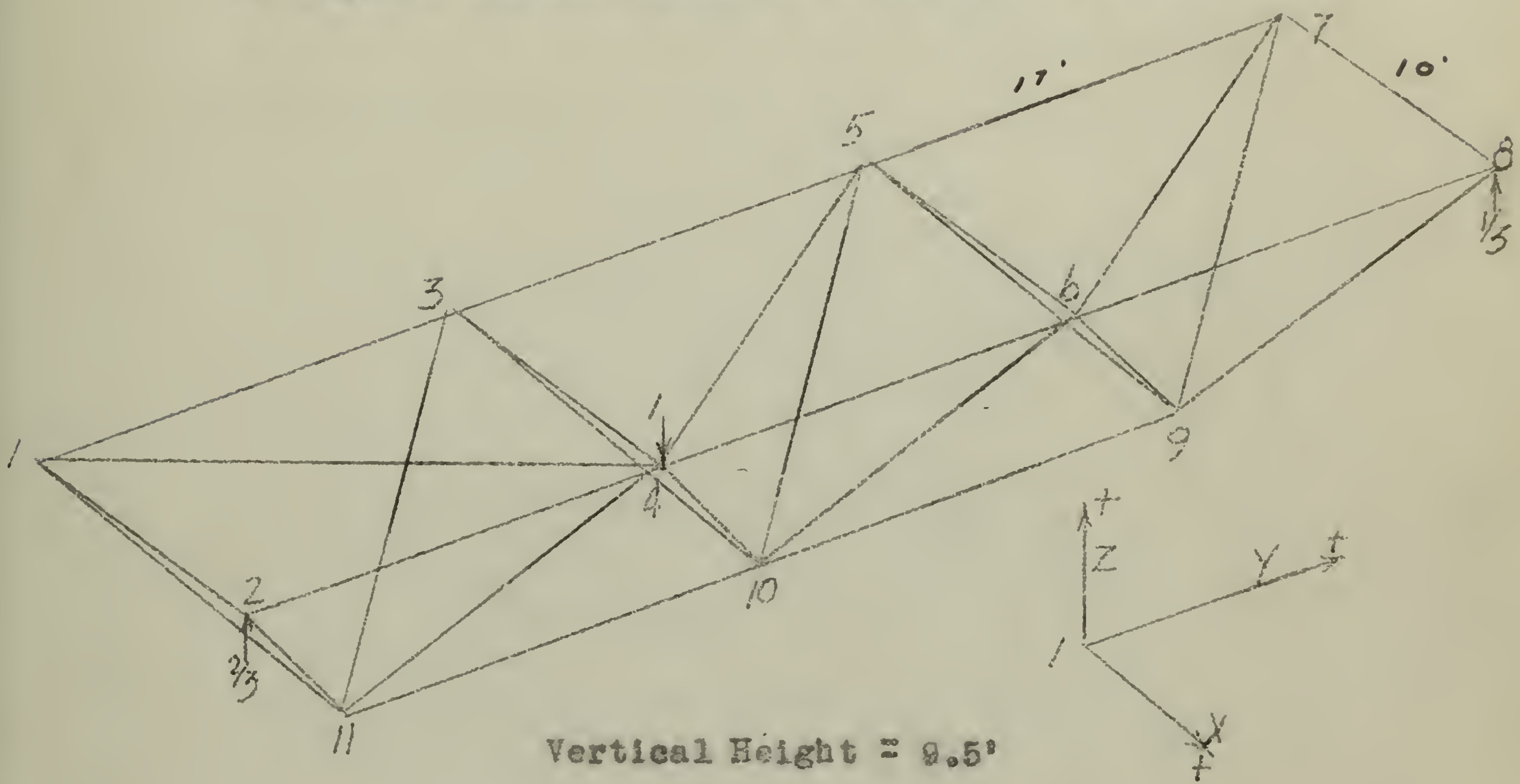
Proof of equation:

$$\text{Stress} = ht \times \frac{L}{h}$$

The proof of this equation can be made by taking a simple joint of a space frame and applying the

method of tension coefficients.

In the following truss, the load of unity acts parallel to one axis and acts on a member connecting two supports. This is, of course, a special loading and for the reactions shown would not be stable under any other loading. It will be a good example, however, to explain the derivation of this method of solution.



Vertical Height = 9.5'

In the proof, instead of writing $t_{BA}(x_A - x_B)$, the term will be $t_{BA}X_{BA}$ with the proper sign on the projection X_{BA}.

Solving for joint 2.

In the X direction:

$$t_{21}(-X_{21}) + t_{2-11}(-X_{2-11}) = 0$$

In the Y direction:

$$t_{24}(Y_{24}) + t_{2-11}(Y_{2-11}) = 0$$

In the Z direction:

$$t_{2-11}(-Z_{2-11}) + 2/3 = 0$$

Since joint 2 is the origin of the solution, the member 2-11 projected into the plane of the force gives the distance h. This projection equals $Z_{2-11} = h$

Therefore:

$$t_{2-11} = \frac{2/3}{h}$$

$$t_{21} = \frac{-X_{2-11}}{X_{21}} \left(\frac{2/3}{h}\right)$$

$$t_{24} = \frac{-Y_{2-11}}{Y_{24}} \left(\frac{2/3}{h}\right)$$

Since every member solved at this joint has some constant over h and every member to be solved depends upon these first three members, it can be seen that

the tension coefficient of every member in the structure will be a constant over h. Therefore, let all tension coefficients be multiplied by h.

This gives:

$$ht_{2-11} = 2/3$$

$$ht_{21} = \frac{-X_{2-11}}{X_{21}} \quad \underline{(2/3)}$$

$$ht_{24} = \frac{-Y_{2-11}}{Y_{24}} \quad (2/3)$$

The ht's now equal a ratio of projections times the reaction (2/3). The ratio is the projection of the known to the projection of the unknown.

In this truss the projection ratios of all members will be 1/1, 1/2, or 2/1. This will not be the case in all structures of course but since space frames usually have some symmetry, the ratios are easily determined.

To go back to the equations:

The X projection ratio $-\frac{5}{10} = -\frac{1}{2}$

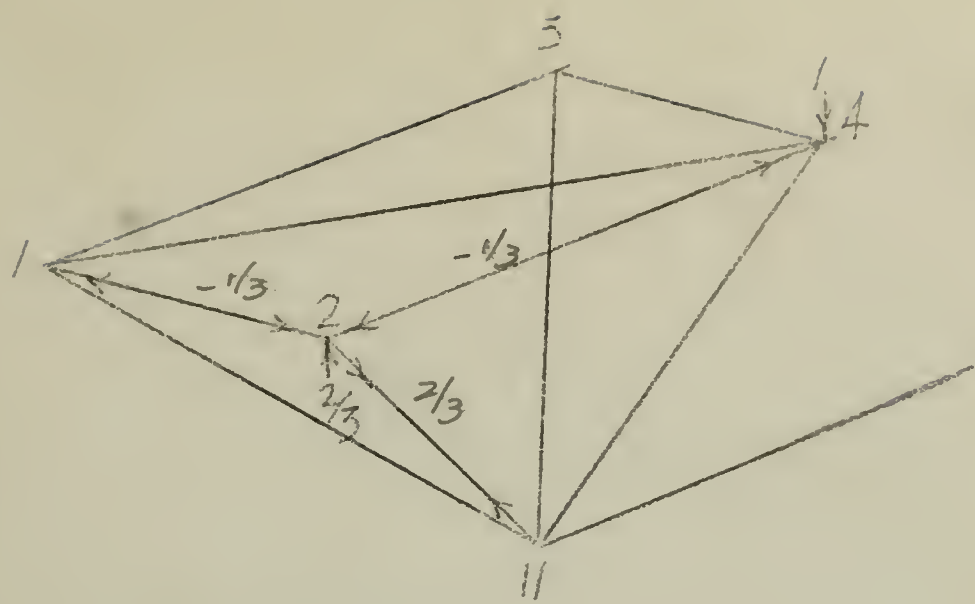
The Y projection ratio $-\frac{8.5}{17} = -\frac{1}{2}$

Therefore:

$$ht_{2-11} = 2/3$$

$$ht_{21} = -1/3$$

$$ht_{24} = -1/3$$



Solution of Joint 1:

In the X direction:

$$t_{12}(X_{12}) + t_{12}(X_{12}) + t_{1-11}(X_{1-11}) = 0$$

In the Y direction:

$$t_{13}(Y_{13}) + t_{14}(Y_{14}) + t_{1-11}(Y_{1-11}) = 0$$

In the Z direction:

$$t_{1-11} = 0$$

From joint 2, $t_{12} = -\frac{1/3}{h}$

Therefore:

$$t_{14} = \frac{X_{12}}{X_{14}} \left(\frac{1/3}{h}\right) = \frac{1}{1} \times \frac{1/3}{h} = \frac{1/3}{h}$$

$$t_{13} = \frac{-Y_{14}}{Y_{13}} \frac{(1/3)}{h} = \frac{-1}{1} \times \frac{1/3}{h} = -\frac{1/3}{h}$$

And,

$$ht_{14} = 1/3$$

$$ht_{13} = -1/3$$

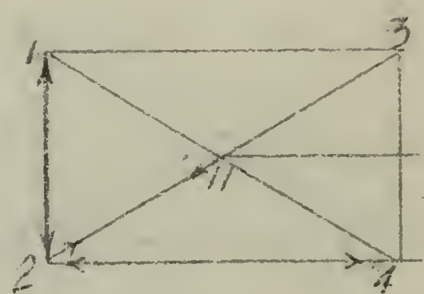
$$ht_{1-11} = 0$$

The solution of ht's thus derived is as tedious as the tension coefficients; however using the few rules stated, and solving the projection ratios mentally, the solution is made on sight for all ht's of the truss. In order to demonstrate, the joints and members of the whole truss will be solved in steps, and an explanation will be made of each step.

Joint 2:

Y-Z Plane

X-Y plane



From Y-Z Plane:

$ht_{2-11} = 2/3$ - the projection ratio is 1/1 when the load lies parallel to one of the axis.

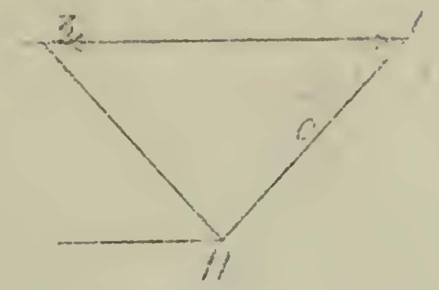
$ht_{24} = -1/3$ - the projection of the known, 2-11 to the unknown 24 is 1/2. $1/2 \times 2/3 = 1/3$. The minus sign shows the compression necessary to take the tension of 2-11

From X-Y Plane:

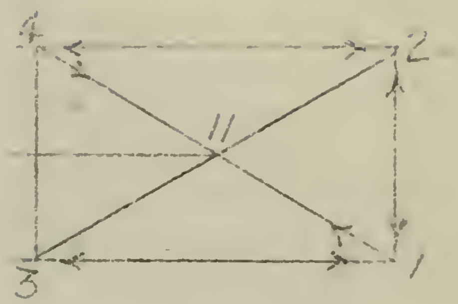
$ht_{21} = -1/3$ - the projection of the known, 2-11, to the unknown 21 is 1/2. 1/2 x the known ht_{2-11} which is $2/3 = 1/3$. Member is in compression.

Joint 1:

Y-Z Plane



X-Y Plane



From Y-Z Plane:

ht_{1-11} = 0 - no reaction or load.

From X-Y Plane:

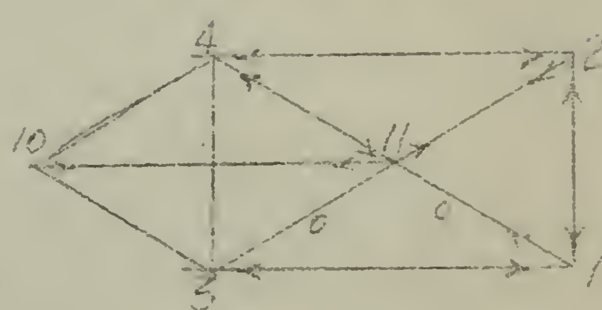
ht_{14} = 1/3 - the projection of the known 12, to the projection of the unknown, 14, is 1/1. 1/1 x the known ht which is 1/3 = 1/3.

ht_{13} = -1/3 - the projection of the known, 14, to the projection of the unknown, 13, is 1/1. 1/1 x the known ht which is 1/3 = 1/3. Member is in compression.

Joint 11:

Y-Z Plane

X-Y Plane



From Y-Z Plane:

ht_{11-3} = 0 - the stress in member 11-1 is 0.

ht_{11-4} = -2/3 - the projection of the known, 11-2, to

the projection of the unknown, 11-4, is 1/1. 1/1 x the known which is 2/3 = 2/3. Member is in compression.

From X-Y Plane:

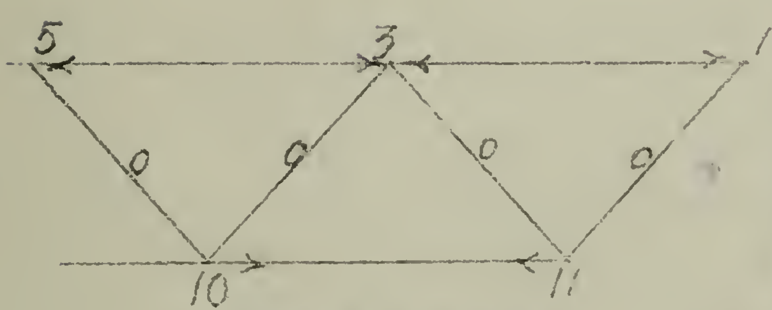
$$ht_{11-10} = 2/3 - \frac{\text{projection known, 11-2}}{\text{projection unknown, 11-10}} = \frac{1}{2}$$

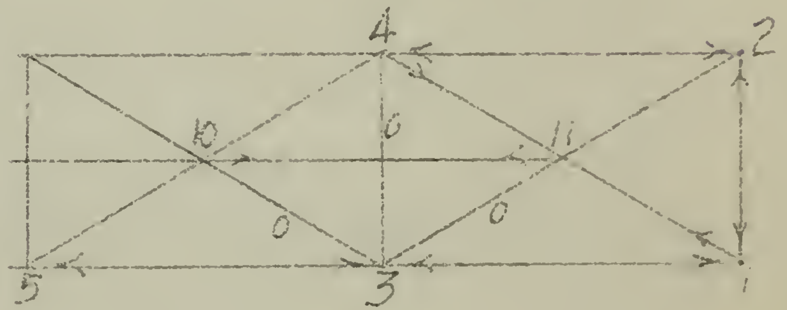
$$\frac{\text{projection known, 11-4}}{\text{projection unknown, 11-10}} = \frac{1}{2}$$

Both members act in same direction; therefore, 1/2 x 2/3 + 1/2 x 2/3 = 2/3.

Joint 3:

Y-Z Plane X-Y Plane





From Y-Z Plane:

$ht_{3-10} = 0 - ht_{3-11} \quad 0$

From X-Y Plane:

$ht_{34} = 0$ - only member at the point which does not lie in plane with other members.

$$ht_{35} = -1/3 - \frac{\text{projection known, 31}}{\text{projection unknown, 35}} = \frac{1}{1}$$

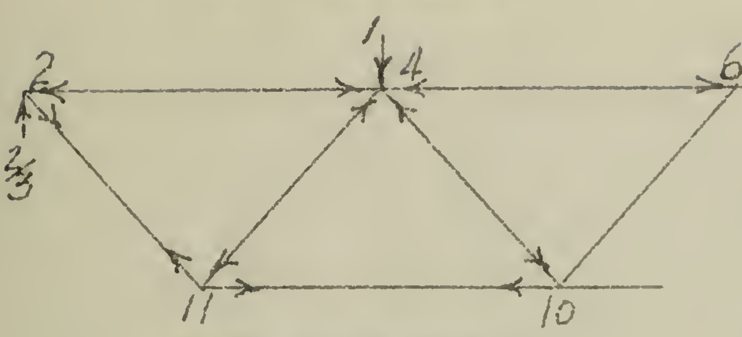
1/1 x 1/3 = 1/3.

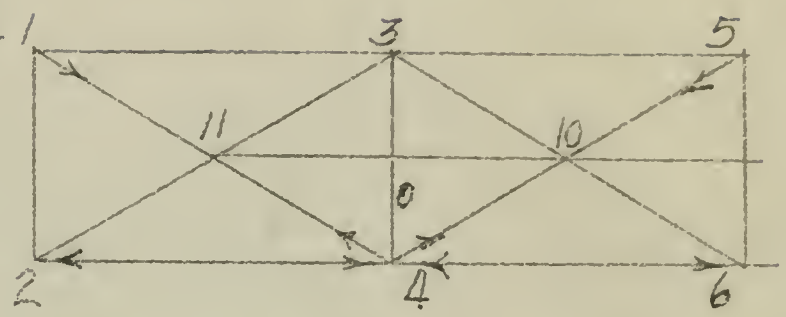
Member is in compression.

Joint 4:

Y-Z Plane

X-Y Plane





From Y-Z Plane:

ht_{4-10} = -1/3 - projection ratio = 1/1. Knowns act in opposite directions; therefore 1 - 2/3 = 1/3.

Member is in compression.

From X-Y Plane: In X direction:

$ht_{45} = 1/6 - \frac{\text{projection 4-11}}{\text{projection 45}} = \frac{1}{2}$

$\frac{\text{projection 4-10}}{\text{projection 45}} = \frac{1}{2}$

$\frac{\text{projection 41}}{\text{projection 45}} = \frac{1}{1}$

$1/2 \times 2/3 + 1/2 \times 1/3 - 1/1 \times 1/3 = 1/6.$

In Y Direction:

$ht_{46} = 1/3 - \frac{\text{projection 42}}{\text{projection 46}} = \frac{1}{1}$

$\frac{\text{projection 4-11}}{\text{projection 46}} = \frac{1}{2}$

$\frac{\text{projection 45}}{\text{projection 46}} = \frac{1}{1}$

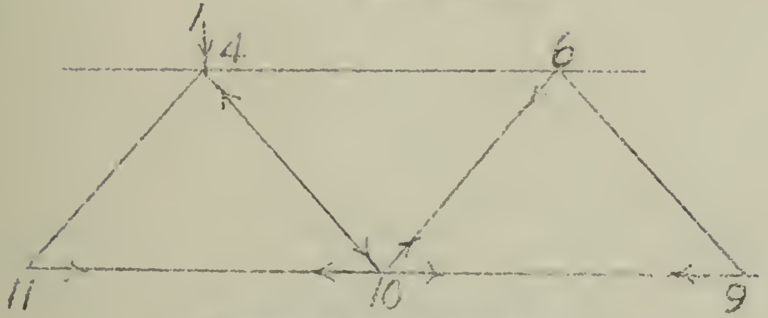
$\frac{\text{projection 4-10}}{\text{projection 46}} = \frac{1}{2}$

$\frac{\text{projection 41}}{\text{projection 46}} = \frac{1}{1}$

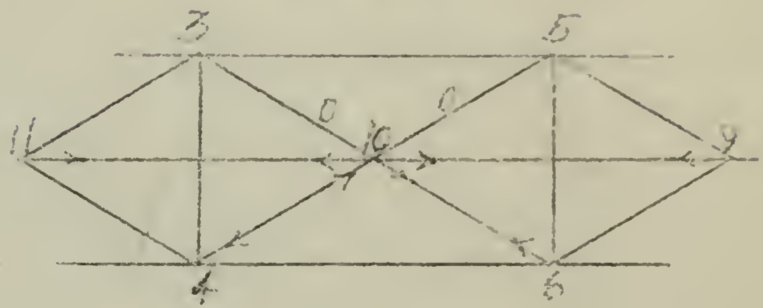
$$1/1 \times 1/3 + 1/2 \times 2/3 + 1/1 \times 1/6 - 1/2 \times 1/3$$
$$- 1/1 \times 1/3 = 1/3$$

Joint 10:

Y-Z Plane



X-Y Plane



From Y-Z Plane:

$ht_{10-5} = 0 - ht_{10-3} = 0$

$ht_{10-6} = 1/3 - \dfrac{\text{projection 10-4}}{\text{projection 10-6}} = \dfrac{1}{1}$

$1/1 \times 1/3 = 1/3.$

From X-Y Plane:

$ht_{10-9} = 1/3 - \dfrac{\text{projection 10-11}}{\text{projection 10-9}} = \dfrac{1}{1}$

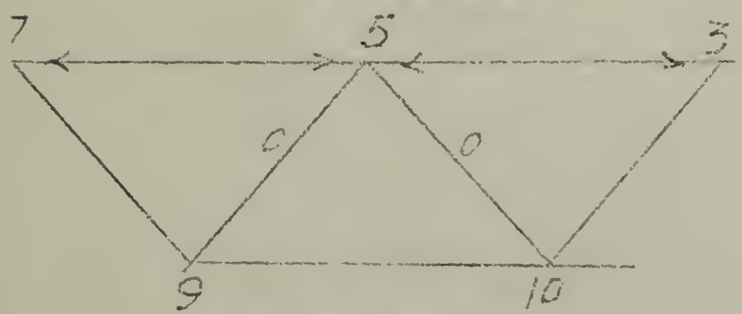
$\dfrac{\text{projection 10-4}}{\text{projection 10-9}} = \dfrac{1}{2}$

$\dfrac{\text{projection 10-6}}{\text{projection 10-9}} = \dfrac{1}{2}$

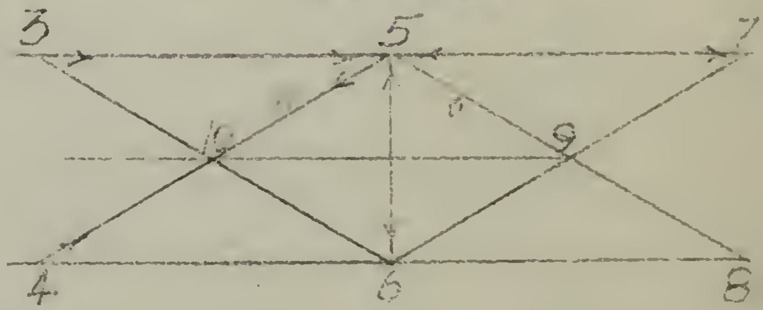
$1/1 \times 2/3 - 1/2 \times 1/3 - 1/2 \times 1/3 = 1/3.$

Joint 5:

Y-Z Plane



X-Y Plane



From Y-Z Plane:

$ht_{59} = 0 - ht_{5-10} = 0$

From X-Y Plane: In X direction:

$$ht_{56} = -1/6 - \frac{\text{projection } 54}{\text{projection } 56} = \frac{1}{1}$$

$$1/1 \times 1/6 = 1/6.$$

Member is in compression

In Y direction:

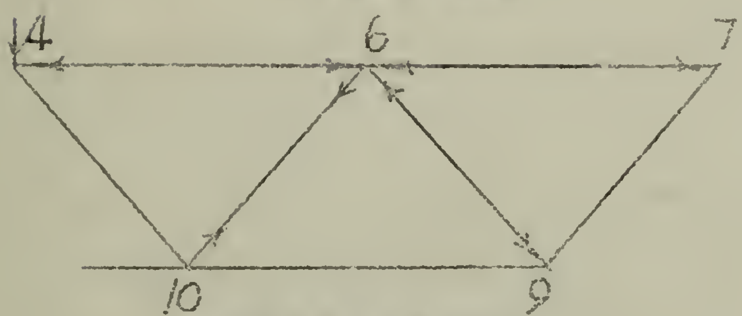
$$ht_{57} = -1/6 - \frac{\text{projection } 53}{\text{projection } 57} = \frac{1}{1}$$

$$\frac{\text{projection } 54}{\text{projection } 57} = \frac{1}{1}$$

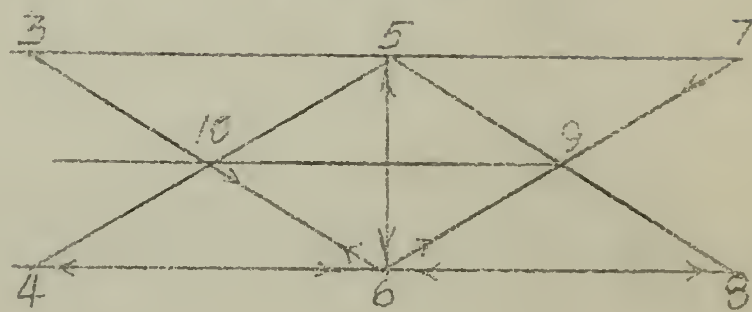
$$1/1 \times 1/3 - 1/1 \times 1/6 = 1/6.$$

Joint 6:

Y-Z Plane



X-Y Plane



From Y-Z Plane:

$$ht_{69} = -1/3 - \frac{\text{projection } 6\text{-}10}{\text{projection } 69} = \frac{1}{1}$$

$$1/1 \times 1/3 = 1/3.$$

Member is in compression.

From X-Y Plane: In X direction:

$$ht_{67} = 1/6 - \frac{\text{projection } 65}{\text{projection } 67} = \frac{1}{1}$$

$$\frac{\text{projection } 69}{\text{projection } 67} = \frac{1}{2}$$

$$\frac{\text{projection } 6\text{-}10}{\text{projection } 67} = \frac{1}{2}$$

X direction, X-Y Plane continued:

$$1/1 \times 1/6 + 1/2 \times 1/3 - 1/2 \times 1/3 = 1/6$$

In Y direction:

$$ht_{68} = -1/6 - \frac{\text{projection } 64}{\text{projection } 68} = \frac{1}{1}$$

$$\frac{\text{projection } 67}{\text{projection } 68} = \frac{1}{1}$$

$$\frac{\text{projection } 69}{\text{projection } 68} = \frac{1}{2}$$

$$\frac{\text{projection } 6\text{-}10}{\text{projection } 68} = \frac{1}{2}$$

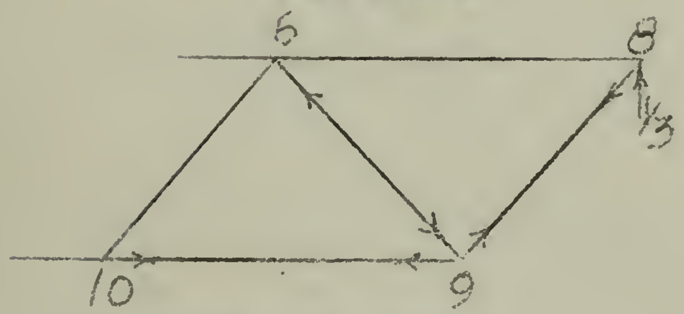
$$1/1 \times 1/3 + 1/1 \times 1/6 - 1/2 \times 1/3$$

$$- 1/2 \times 1/3 = 1/6$$

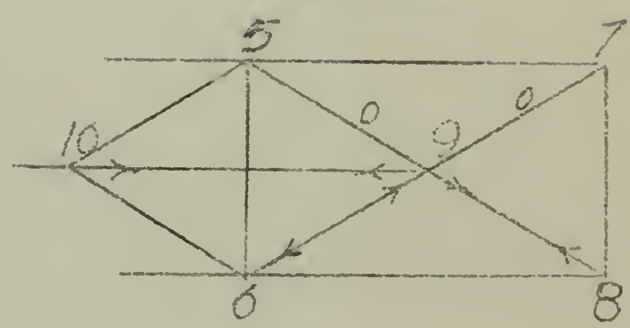
Member is in compression

Joint 9.

Y-Z Plane



X-Y Plane



From Y-Z Plane;

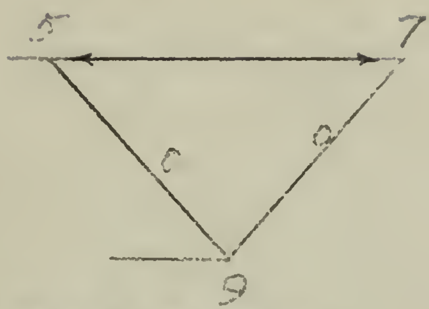
$$ht_{97} = 0 - ht_{95} = 0$$

$$ht_{98} = 1/3 - \frac{\text{projection } 96}{\text{projection } 98} = \frac{1}{1}$$

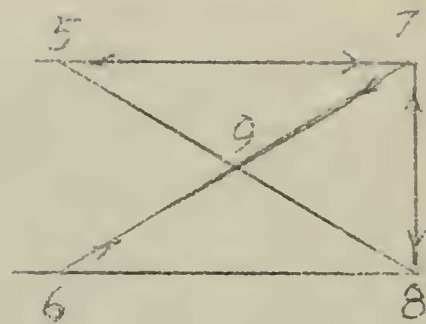
$$1/1 \times 1/3 = 1/3.$$

Joint 7:

Y-Z Plane



X-Y Plane

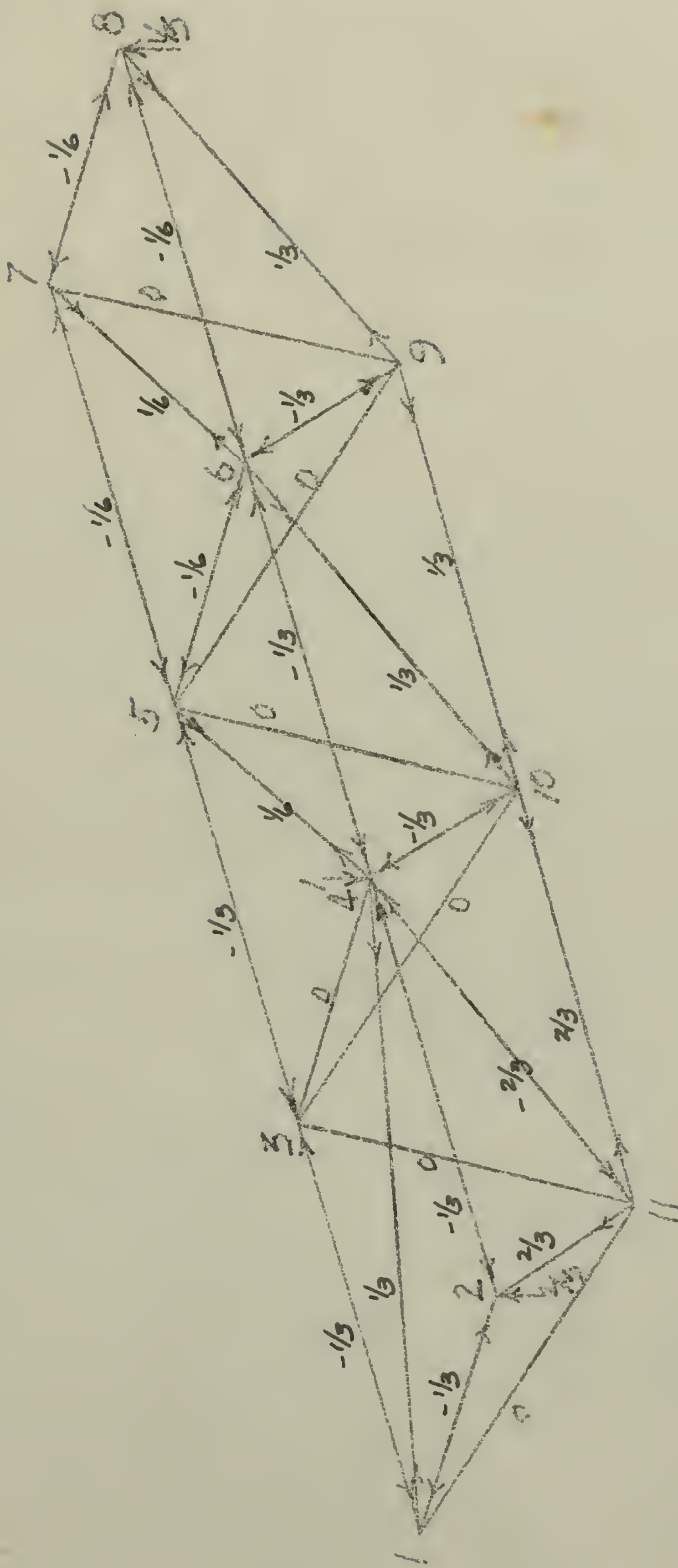


From X-Y Plane:

$ht_{78} = -1/6 - \frac{\text{projection 76}}{\text{projection 78}} = \frac{1}{1}$

$1/1 \times 1/6 = 1/6.$

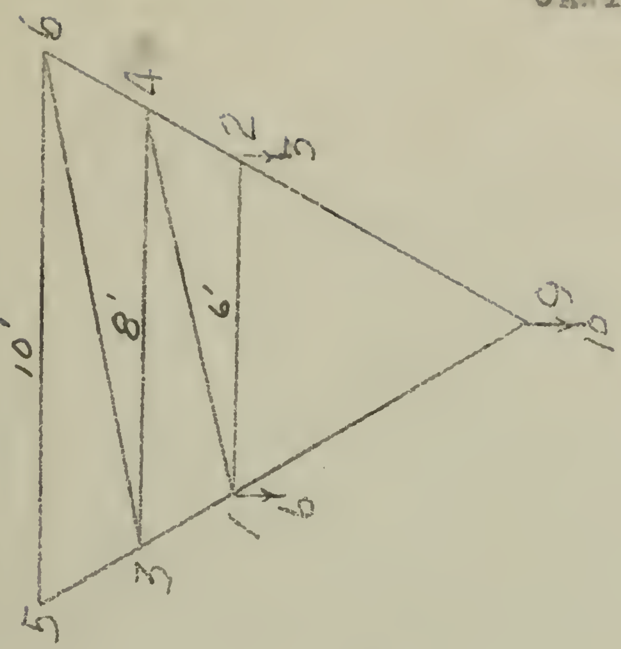
Member is in compression.

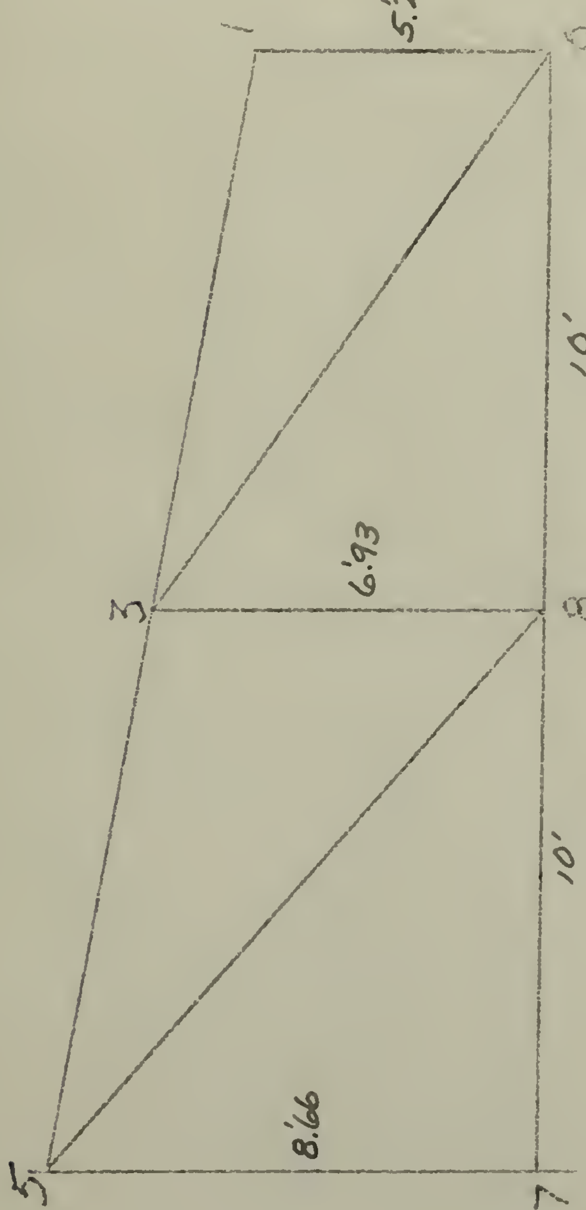


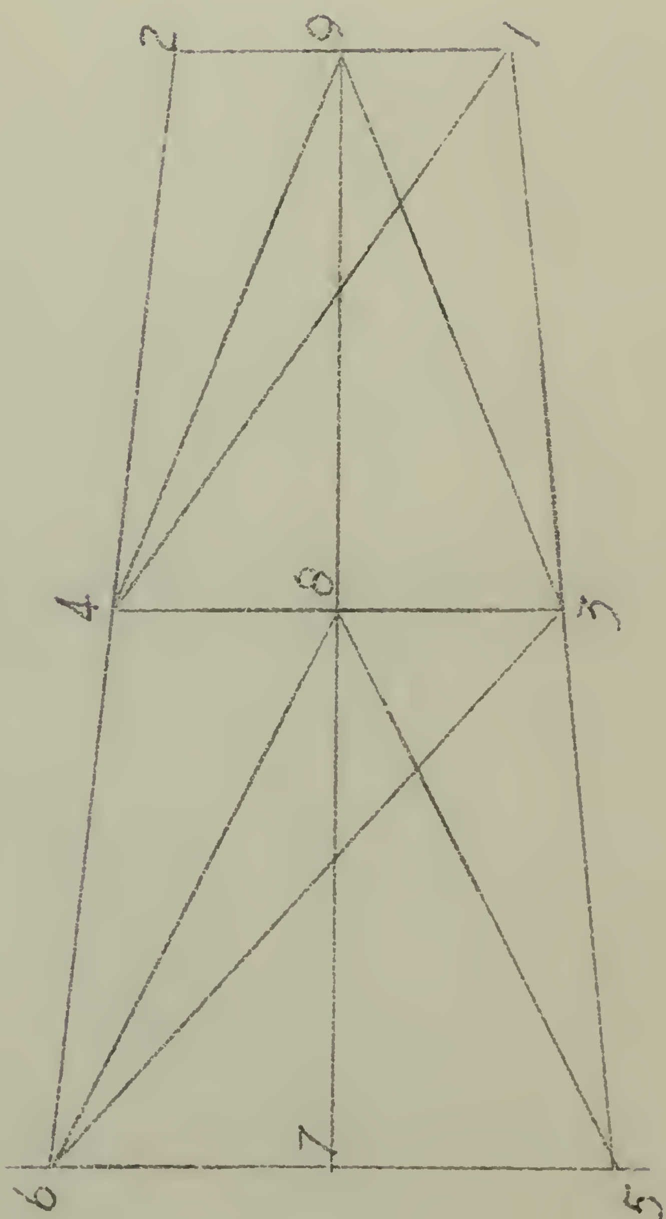
Summary table:

Member	Length (L)	ht number	h	Stress ht x $\frac{L}{h}$
12	10	-1/3	9.5	-.35
13	17	-1/3	9.5	-.595
14	19.7	1/3	9.5	.691
1-11	13.7	0	9.5	.000
24	17	-1/3	9.5	-.595
2-11	13.7	2/3	9.5	.963
34	10	0	9.5	.000
35	17	-1/3	9.5	-.595
3-11	13.7	0	9.5	.000
3-10	13.7	0	9.5	.000
45	19.7	1/6	9.5	.350
46	17	-1/3	9.5	-.595
4-11	13.7	-2/3	9.5	-.963
4-10	13.7	-1/3	9.5	-.481
56	10	-1/6	9.5	-.175
57	17	-1/6	9.5	-.298
5-10	13.7	0	9.5	.000
59	13.7	0	9.5	.000
67	19.7	1/6	9.5	.345
68	17	-1/6	9.5	-.298
6-10	13.7	1/3	9.5	.481
69	13.7	-1/3	9.5	-.481
78	10	-1/6	9.5	-.175
79	13.7	0	9.5	.000
89	13.7	1/3	9.5	.481
9-10	17	1/3	9.5	.595
10-11	17	2/3	9.5	1.215

CANTILEVER TRUSS





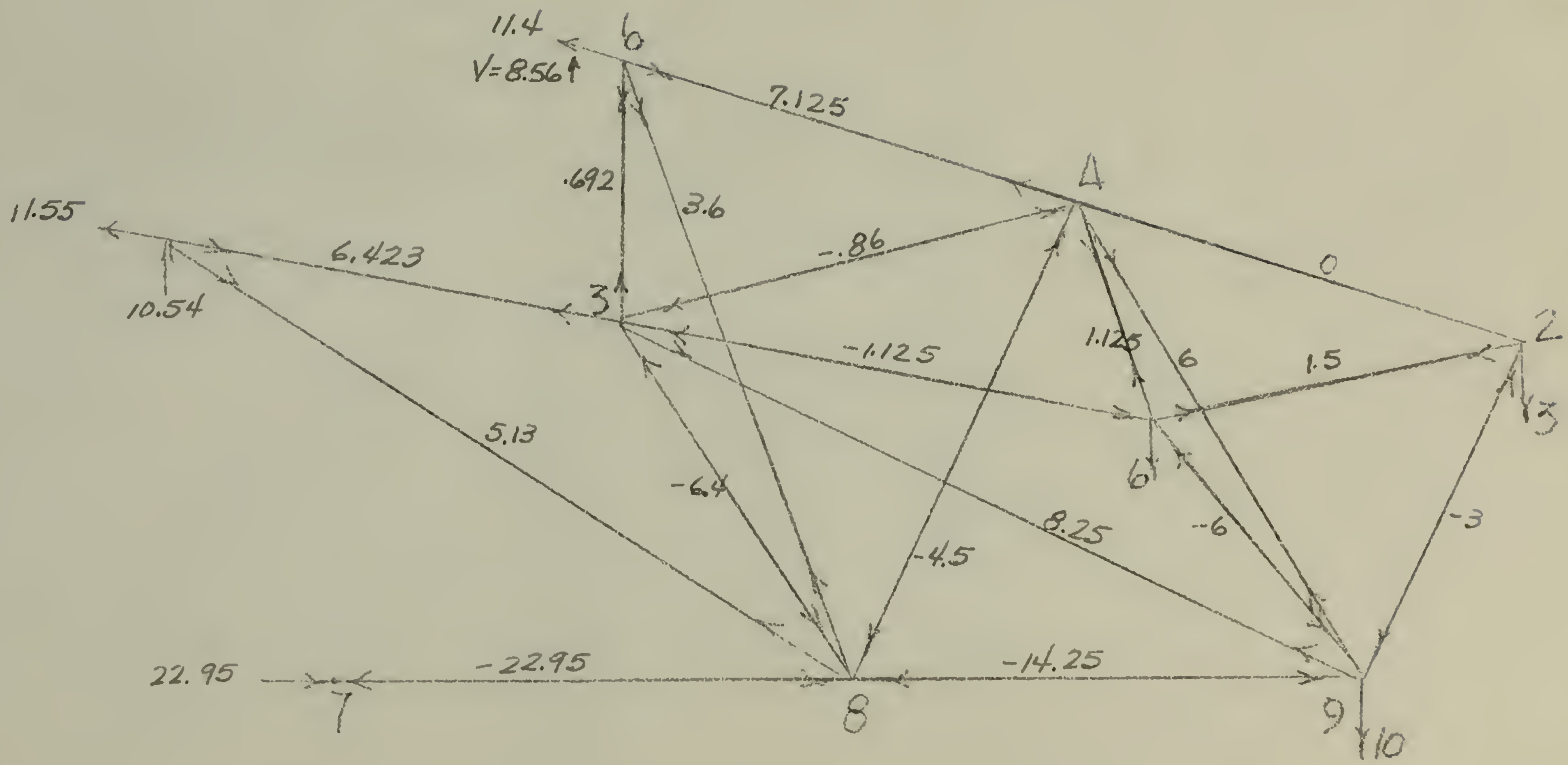


Cantilever truss.

The solution of this truss was started at joint 2 instead of at one of the reactions. The distance h is equal to 5.2 feet since the distance from joint 2 to joint 9 parallel to the load line is 5.2 feet. Since the truss does not have a constant vertical cross section, the vertical reactions are not direct sums of the vertical projections of the ht numbers. To explain the reason for this, suppose that the solution had started at reaction point 5. First, it would have been necessary to solve for the reactions then the regular solution could have been started. The h distance for this solution would be the vertical depth of the truss at the reactions. If the solution is started at joint 2, therefore, the vertical reactions at 5 and 6 would be:

$$\frac{\text{vertical depth truss at reactions}}{h} \times \text{sum of ht numbers.}$$

The solution of this cantilever follows.

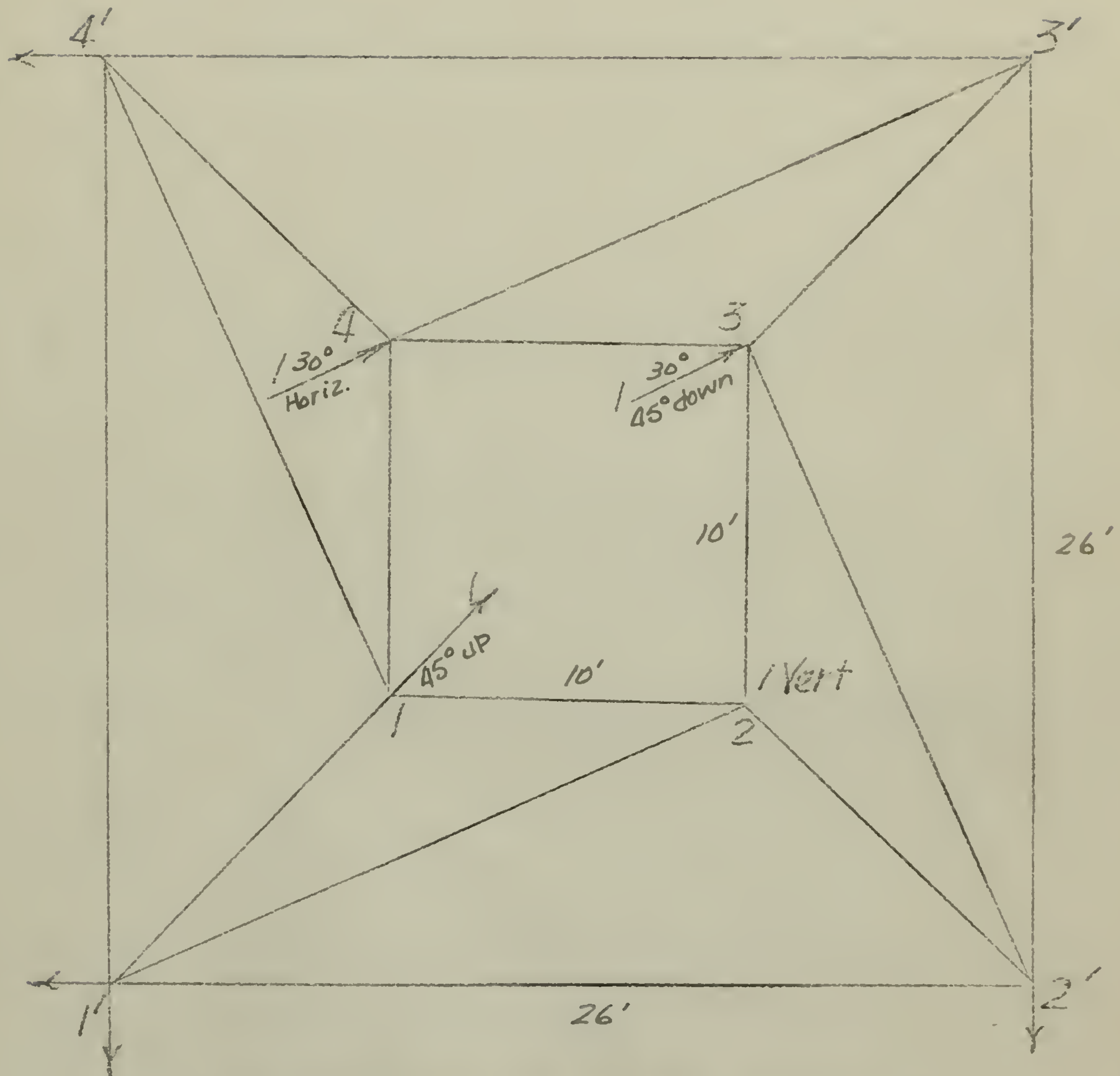


11.4
6
V=8.56
7.125
.692
3.6
4
11.55
6.423
-.86
0
3
10.54
-1.125
1.125
6
1.5
2
5
5.13
6
-6.4
8.25
-6
-3
-4.5
22.95
-22.95
-14.25
7
8
9
10

Summary table:

Member	Length (L)	bt number	b	Stress $bt \times \frac{L}{b}$
91	6.0	-6.000	5.2	-6.91
92	6.0	-3.000	5.2	-3.46
93	12.8	8.250	5.2	20.3
98	10.0	-14.250	5.2	-27.45
94	12.8	6.000	5.2	14.75
13	10.2	-1.125	5.2	-2.18
12	6.0	1.500	5.2	1.73
14	12.3	1.125	5.2	2.63
24	10.2	0.000	5.2	0.00
83	8.0	-6.400	5.2	-9.83
87	10.0	-22.950	5.2	-44.10
85	14.14	5.130	5.2	13.85
86	14.14	3.600	5.2	9.81
84	8.0	-4.500	5.2	-6.92
35	10.2	6.423	5.2	12.60
36	13.56	.692	5.2	1.80
34	8.0	-.860	5.2	-1.33
46	10.2	7.125	5.2	14.02

PEDESTAL



Vertical reactions at each support

Vertical height = 20'

Pedestal.

With this type loading, there is no way of picking the distance h from any of the rules given previously. If the solution is started at joint 1, the load can be split into $\sqrt{2}$ acting vertically, 1/2 in the X direction, and 1/2 in the Y direction. Suppose that the solution is started at one of the reaction joints. The distance h would then be 20 feet. Using this distance for h, the ht numbers of each member at each joint would be multiplied by $\frac{\text{projection of member}}{h}$ to set up the force balance equations at the joint if the solution were started at a loaded joint, such as joint 1.

For example, the equations at joint 1 will be:

In the Z direction:

$$\sqrt{2} \quad ht_{11'} - ht_{14'}$$

In the Y direction:

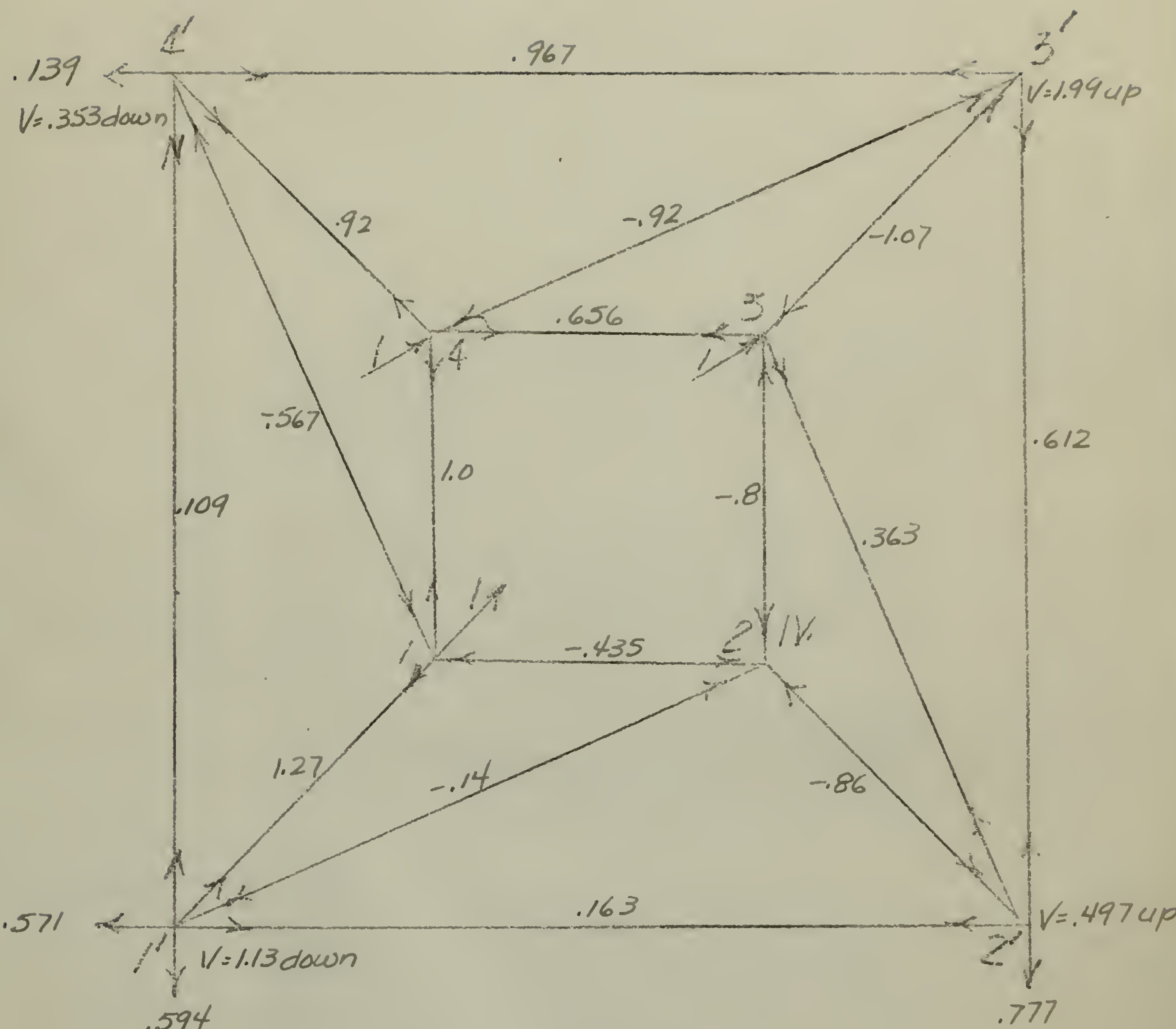
$$\tfrac{1}{2} = 8/20\ ht_{11'} - 8/20\ ht_{14'} + 10/20\ ht_{12}$$

In the X direction:

$$\tfrac{1}{2} = 8/20\ ht_{11'} + 18/20\ ht_{14'} - 10/20\ ht_{14}$$

This is the reverse of the procedure used in the former problem when the vertical reactions were determined.

The solution for the pedestal follows.



.967
.139
V=.353 down
V=1.99 up
.92
-.92
-1.07
.656
4
3
-.567
.612
1.0
.109
-.8
.363
-.435
1.27
-.14
-.86
.571
.163
V=.497 up
V=1.13 down
.594
.777

Summary table:

Member	Length (L)	ht number	h	Stress $ht \times \frac{L}{h}$
12	10	-.435	20	-.218
11'	23	1.27	20	1.455
14'	27.7	-.567	20	-.786
14	10	1.000	20	.500
21'	27.7	-.14	20	-.194
22'	23	-.86	20	-.995
23	10	-.80	20	-.400
32'	27.7	.363	20	.503
33'	23	-1.07	20	-1.228
34	10	.656	20	.328
43'	27.7	-.92	20	-1.275
44'	23	.92	20	1.055
1'2'	26	.163	20	.199
2'3'	26	.612	20	.795
3'4'	26	.967	20	1.256
4'1'	26	.109	20	.142

SUMMARY

1. The simplified method is a system similar to index numbers in planar trusses. These numbers are called ht numbers in this paper.
2. The ht numbers are multiplied by $\frac{\text{Length}}{h}$ to give the stress in the members.
3. The h distance is the distance from the first joint solved to the next joint on a line parallel to the load line, unless the load line is not parallel to one of the axes. In that case, the distance h is the vertical distance from the first joint solved to the next joint and the ht numbers must be multiplied by the ratio $\frac{\text{length of member}}{h}$ when the force balance equations are set up at the joints.
4. Reactions are sums of ht number projections unless cross sections parallel to h have changed. In that case, the ht number projections are multiplied by $\frac{\text{cross section distance parallel to h.}}{h}$

CONCLUSIONS

The author of this paper has compared this method with several simplified solutions and in the problems solved, this solution has been the simpler. It is admitted that the solution has rules which are cumbersome to handle; however, if a person has several space frames to solve, it would be beneficial to investigate with this method. This method would be especially advantageous if solving for influence lines for three chord bridge trusses. It was hoped that it could be used for solving the bar ring stresses in the Schwedler dome, but the addition of stresses around the ring could not be handled. The dome structure itself is easily solved by the method explained in this paper.

Further research of this subject may bring further simplifications.

BIBLIOGRAPHY

Charles M. Spofford	Theory of Structures
Linton E. Grinter	Theory of Modern Steel Structures, Vol. II
F. H. Constant	Proceedings, A. S. C. E. May, 1943
F. H. Constant	Civil Engineering December, 1935
A. J. S. Pippard	Transactions, A.S.C.E. 1935

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